PHASE RETRIEVAL FROM TWO DEFOCUSED IMAGES BY FAST FOURIER TRANSFORM AND SYMMETRY CRITERIA APPLIED TO THE TRANSPORT-OF-INTENSITY EQUATION SOLUTION

V.V. Volkov (volkov@bnl.gov), and Y. Zhu
Materials Science Division, Brookhaven National Laboratory, Upton, NY 11973

The problem of phase retrieval from intensity measurements plays an important role in many fields of physical research: optics, electron and x-ray microscopy, crystallography and others. One of possible approaches to solving this problem is the use of transport-of-intensity equation (TIE) introduced originally by Teague [1]. We have successfully demonstrated the practical use of TIE to materials problem solving, including magnetic inductance mapping. However, this approach still needs further improvements, since the results appear to be sensitive to many experimental parameters. The TIE is derived in paraxial (Fresnel) wave approximation and utilizes a relation of z-gradient for intensity distribution I(x,y) to phase distribution ϕ(x,y) in imaging plane via the 2nd order elliptic differential equation

$$-k f_z I = - ?(I \quad \varphi), \qquad (1) \qquad\qquad f_z \int_\Omega I_z(r)\,dr = 0 \qquad (2)$$

with boundary conditions I(r)>0 inside Ω image and I=0 on and outside δΩ=Γ area. It is noted [1] that for Eq.(1) the law of energy conservation (Eq.(2)) must hold. In the present work we show that boundary condition I(Γ) = 0 in terms of Dirichlet-Neumann boundary problem follows from Eq.(2) and therefore it is not necessary to postulate. By applying the Stoke's theorem we found another equivalent formulation of the boundary problem suitable for a quick solution of Eq.(1) via fast Fourier transform (FFT). Therefore we suggest a new correct solution of the problem (1)-(2) by imposing a necessary symmetry condition derived from Eq.(2) for auxiliary function as $\nabla\Psi = I\nabla\varphi = even$, which dramatically reduces a general solution for recovered phase as $\varphi = \varphi(FFT) + H$, where harmonic function obeys $\nabla^2 H=0$. The calculation should be done in the domain area 2x2Ω with symmetry condition $\varphi = even$. This condition reduces harmonic function ideally down to term $A+E(x^2-y^2)$, where A and E are the only 2 unknown fitting parameters. Meanwhile this approach allows to get a non-spoiled solution of the recovered phase φ (x,y) in the entire Ω image area when compared to traditional "zero" masking approach and associated wrap-around problem caused by the FFT solution of Eq.(1). The successful applications of the new approach to the phase retrieval problem will be demonstrated for both non-magnetic and magnetic materials.

References:

1. M.E. Teague, J. Opt. Soc. Am.**73**(11), 1434-1441 (1983).

2. Work supported by US DOE, DE-AC02-98CH10886. Stimulating discussions and partial use of the computer codes from M. De Graef are also acknowledged.